Hypergraph states, their entanglement and robustness properties

Mariami Gachechiladze, Nikoloz Tsimakuridze, Costantino Budroni, Otfried Gühne

Quantum Physics and Logic, Glasgow 2016

Phys. Rev. Lett. 116, 070401 (2016)

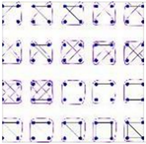
Motivation

Extreme violation of local realism in quantum hypergraph states
Phys. Rev. Lett. 116, 070401 (2016)

Motivation

Sarah Q. Malone
March 4 · Flipboard ·

Does this mean that there are some places we're free to be wildly imaginative? 🙂



Physicists find extreme violation of local realism in quantum hypergraph states

In the new study, the physicists discovered that quantum hypergraph states have perfect…

FLIP.IT

Gillian Whittle — 3 Comments

Like — Share

Gillian Whittle I got tangled up here...
Like · March 4 at 11:39pm

Terry Robson We've heard about redneck states but which are the hypergraph states? Are they less extreme or more in their violation of the locality?
Like · March 5 at 1:01pm · Edited

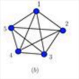
David DuVal
March 4 ·

Great news! In a new study, physicists discovered that quantum hypergraph states have perfect correlations that are highly nonlocal. This means that hypergraph states strongly violate local realism. Got it?

3 — 2 Comments

Like — Share

David DuVal http://m.phys.org/.../2016-03-physicists-extreme...



Physicists find extreme violation of local realism in quantum hypergraph states

(Phys.org)—Many quantum technologies rely on…

PHYS.ORG

Like · March 4 at 10:11pm

Kristina Swanson Junior, when we get home, remind me to punch your mama right in the mouth. 🙂
Like · March 4 at 11:21pm

[–] **LeQuint_Dickey** 18 points 1 month ago
Is there an English version of this article somewhere?

[–] **krubo** 2 points 1 month ago
Does it mean we're going to get ansibles?

[–] **Ketrix** 1 point 1 month ago
So is this basically just a significant step in how we (by we I mean physics scientists) understand matter?
permalink parent

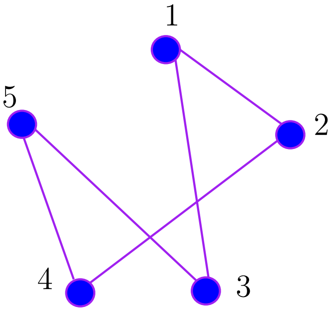
Motivation


FUN TIMES 2016/02/26
Physicists find extreme violation of local realism
Science
Latest News
Thai beer on tap

Content

- Graph States vs Hypergraph States
- Local stabilizer vs nonlocal stabilizer
- Entanglement & Robustness
- Local Pauli and Clifford equivalence rules
- Outlook

What are graph states and why to study them?

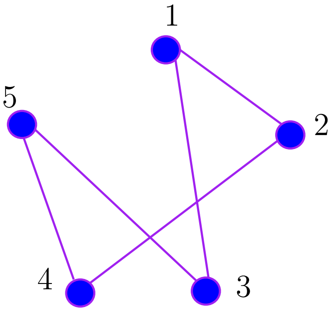


A graph: $G = \{V, E\}$.

Nodes: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

Edges: $C_{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

A graph state: $$|G\rangle = \prod_{e \in E} C_e \, |+\rangle^{\otimes N}.$$



Graph State:
$|G\rangle = \prod_{e\in E} C_e |+\rangle^{\otimes N}$.

Stabiliser States:

$g_1 = X_1 Z_2 Z_3$
$g_2 = X_2 Z_1 Z_4$
$g_3 = X_3 Z_1 Z_5$
$\vdots$

$$g_i |G\rangle = +|G\rangle.$$

Measurement-based quantum computation on cluster states

Robert Raussendorf, Daniel E. Browne,* and Hans J. Briegel
Theoretische Physik, Ludwig-Maximilians-Universität München, Germany
(Dated: February 1, 2008)

We give a detailed account of the one-way quantum computer, a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. We prove its universality, describe why its underlying computational model is different from the network model of quantum computation and relate quantum algorithms to mathematical graphs. Further we investigate the scaling of required resources and give a number of examples for circuits of practical interest such as the circuit for quantum Fourier transformation and for the quantum adder. Finally, we describe computation with clusters of finite size.

Applications

Computational power of correlations

Janet Anders* and Dan E. Browne†
Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom.
(Dated: February 5, 2009)

We study the intrinsic computational power of correlations exploited in measurement-based quantum computation. By defining a general framework the meaning of the computational power of correlations is made precise. This leads to a notion of resource states for measurement-based *classical* computation. Surprisingly, the Greenberger-Horne-Zeilinger and Clauser-Horne-Shimony-Holt problems emerge as optimal examples. Our work exposes an intriguing relationship between the violation of local realistic models and the computational power of entangled resource states.

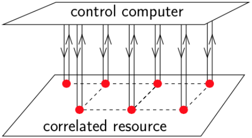


FIG. 1: The control computer provides one of k choices as the classical input (downward arrows) to each of the correlated parties (circles in the resource) and receives one of l choices as the output.

Applications

4·3. *Quantum error correcting codes.* – For *quantum error correcting codes* based on stabilizer codes [22] the *codewords* as well as the *encoding procedures* can be represented as graphs [40, 55, 56]. The latter can be understood along the lines of the previous subsection, because the graph state is the computational resource for implementing the encoding process in terms of the QC_G model. The graphs depicted in fig. 11 for example correspond to the encoding procedures for the five-qubit *Steane code* and the *concatenated* $[7, 1, 3]$-*CSS-code* that encode a state on one qubit into some state on five and 49 qubits respectively.

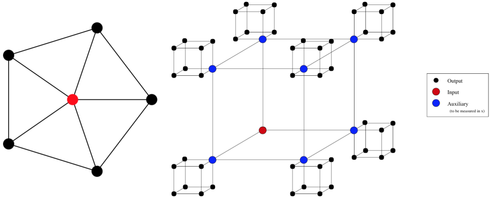
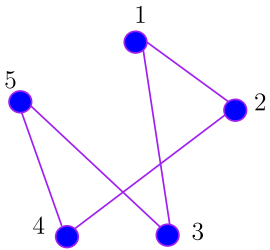


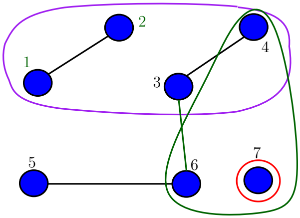
Fig. 11. – –*Five-Qubit-Code and concatenated CSS-Code*– The graphs representing the encoding procedure for the five-qubit and the concatenated $[7, 1, 3]$-CSS-code with input (red), auxiliary (blue) and output (black) vertices.

M.Hein, W.Dür, J. Eisert, R. Raussendorf, M Van Den Nest, H.J. Briegel, Entanglement in Graph States and its Applications. (2006)

Hypergraph States

Graph States vs Hypergraph States





Edges: $C_e = 1 - 2\,|11\ldots1\rangle\langle11\ldots1|$

Edges: $C_{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$

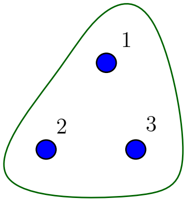
$$C_{abc} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

$$|G\rangle = \prod_{e\in E} C_e \,|+\rangle^{\otimes N}$$

$$|H\rangle = \prod_{e\in E} C_e \,|+\rangle^{\otimes N}$$

Example– Hypergraph States

The simplest HG state look as follows:



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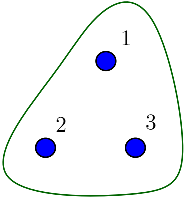
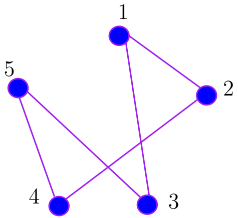


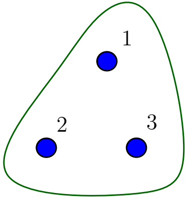
$$|H\rangle = C_{123}\,|+\rangle^{\otimes^3}$$

$$= \frac{1}{\sqrt{8}}\left[\,|000\rangle + |001\rangle + |010\rangle + |100\rangle + |011\rangle + |101\rangle + |110\rangle - |111\rangle\,\right]$$

Graph States vs Hypergraph States

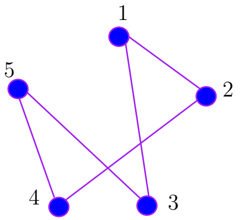


$$|G\rangle = \prod_{e \in E} C_e \, |+\rangle^{\otimes N}$$



$$|H\rangle = \prod_{e \in E} C_e \, |+\rangle^{\otimes N} = C_{123} \, |+\rangle^{\otimes N}$$

Graph States vs Hypergraph States



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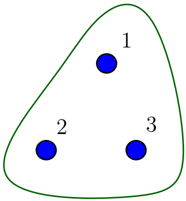

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Stabiliser States:

$h_1 = X_1 C_{23}$
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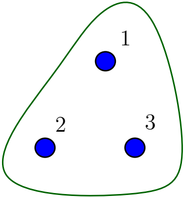
$h_3 = X_3 C_{12}$

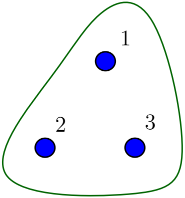
$$h_i |H\rangle = +|H\rangle.$$



$$h_1 = X_1 C_{23}$$
$$h_2 = X_2 C_{13}$$
$$h_3 = X_3 C_{12}$$

$$h_i |H\rangle = + |H\rangle.$$

$$h_1 = X_1 \otimes \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



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$$h_1 = X_1 C_{23}$$
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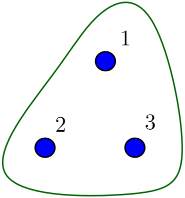
$$h_3 = X_3 C_{12}$$

$$P(+--|XZZ) = 0$$

$$h_i \left|H\right\rangle = + \left|H\right\rangle.$$

Nonlocality proofs for HG states



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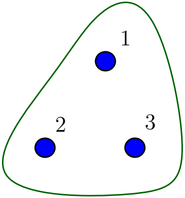
$$P(+--|XZZ) = 0$$

$$P(-+-|XZZ) = ?$$

$$P(--+|XZZ) = ?$$

$$P(-++|XZZ) = ?$$



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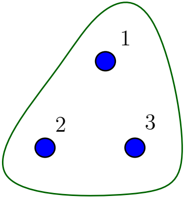


$$h_1 = X_1 C_{23}$$
$$h_2 = X_2 C_{13}$$
$$h_3 = X_3 C_{12}$$

$$h_i \left|H\right\rangle = + \left|H\right\rangle.$$

$$P(+--|XZZ) = 0$$

$$P(-+-|XZZ) = 0$$
$$P(--+|XZZ) = 0$$
$$P(-++|XZZ) = 0$$

$+$ permutations from h_2 and h_3.

Hardy argument for H

For the deterministic assignment of probabilities:

$$P(+--|XZZ) = 0$$

$$P(-+-|XZZ) + P(--+|XZZ) + P(-++|XZZ) = 0$$

$$+$$

permutations coming from h_2, h_3

$$\Rightarrow P(+--|XXX) = 0$$

Hardy argument for H

For the deterministic assignment of probabilities:

$$P(+--|XZZ)=0$$

$$P(-+-|XZZ)+P(--+|XZZ)+P(-++|XZZ)=0$$

$$+$$

permutations coming from h_2, h_3

$$\Rightarrow P(+--|XXX)=0$$

An actual value : $P(+--|XXX)=\frac{1}{16}$.

$|H\rangle = \frac{1}{2}\left[\,|000\rangle + |010\rangle + |100\rangle + |111\rangle\,\right]$ S. Abramsky, C. Constantin, EPTCS 171, 2014, pp. 10-25

Local Hidden Variable Model $1|23$ for H

$$P(+--|XZZ) = 0$$

$$P(-+-|XZZ) + P(--+|XZZ) + P(-++|XZZ) = 0$$

$$+$$

1 local $+$ 2,3 no-signaling

$$+$$

permutations coming from h_2, h_3

Local Hidden Variable Model 1|23 for H

$$P(+--|XZZ) = 0$$

$$P(-+-|XZZ) + P(--+|XZZ) + P(-++|XZZ) = 0$$

$$+$$

1 local $+$ 2,3 no-signaling

$$+$$

permutations coming from h_2, h_3

$$\Rightarrow$$

$P(---|XXX) = \frac{1}{16}$ & $P(---|ZZZ) = \frac{1}{8}$ is banned!

Local Hidden Variable Model $1|23$ for H

$$P(+--|XZZ) = 0$$

$$P(-+-|XZZ) + P(--+|XZZ) + P(-++|XZZ) = 0$$

$$+$$

1 local + 2,3 no-signaling

$$+$$

permutations coming from h_2, h_3

$$\Rightarrow$$

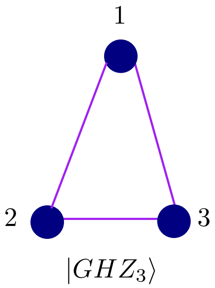
$$P(---|XXX) = \frac{1}{16} \quad \& \quad P(---|ZZZ) = \frac{1}{8} \text{ is banned!}$$

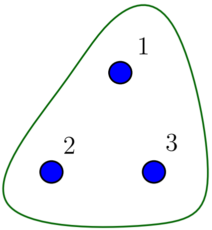
Bell inequality:

$$P(+--|XZZ) + \text{permutations} + P(-R3|XZZ) + \text{permutations} - P(---|ZZZ) + P(---|XXX) \geq 0$$

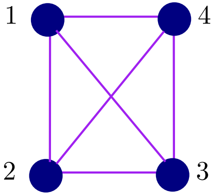
Exponentially growing violation in graph states and hypergraph states

Mermin's Inequalities

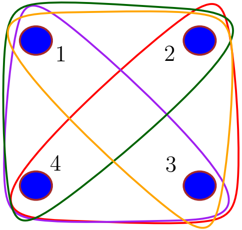




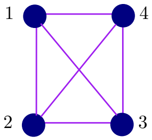
Mermin's Inequalities



$$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle$$



$$|H_4^3\rangle = C_{123}C_{124}C_{134}C_{134}\,|+\rangle^{\otimes 4}$$



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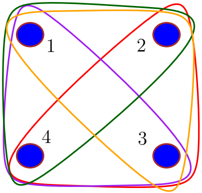
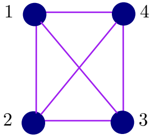


$|H_4^3\rangle = C_{123}C_{124}C_{134}C_{134}\,|+\rangle^{\otimes 4}$

Now for an arbitrary $|GHZ_N\rangle$:

Now for an arbitrary $\left|H_N^3\right\rangle$:

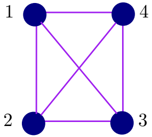
Mermin's Inequalities



$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$

Now for an arbitrary $|GHZ_N\rangle$:

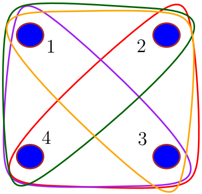
$$
\begin{aligned}
\mathcal{B}_M \quad = \quad & (XZZ\ldots Z \quad + \quad \text{permutations})- \\
& -(XXXZZ\ldots Z \quad + \quad \text{permutations})+ \\
& +(XXXXXZZ\ldots Z \quad + \quad \text{permutations})- \\
& -(\ldots\ldots)
\end{aligned}
$$



$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle$

Now for an arbitrary $|GHZ_N\rangle$:

$$\langle \mathcal{B}_Q \rangle = 2^{N-1} \quad \text{GHZ state}$$
$$\langle \mathcal{B}_C \rangle = 2^{\lfloor N \backslash 2 \rfloor}$$

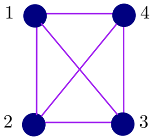


$|H_4^3\rangle = C_{123}C_{124}C_{134}C_{134}\,|+\rangle^{\otimes 4}$

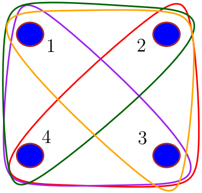
Now for an arbitrary $\left|H_N^3\right\rangle$:

N. David Mermin (1990) Phys. Rev. Lett. 65(15).

Mermin's Inequalities



$|GHZ_4\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$



$|H_4^3\rangle = C_{123}C_{124}C_{134}C_{134}\,|+\rangle^{\otimes 4}$

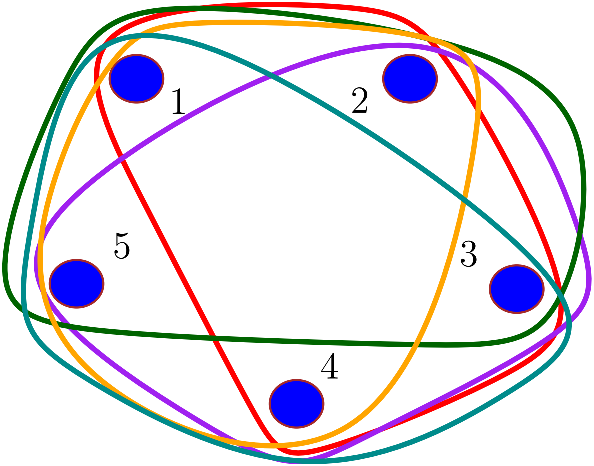
Now for an arbitrary $|GHZ_N\rangle$:

$$\begin{aligned} \langle \mathcal{B}_Q \rangle &= 2^{N-1} \quad \text{GHZ state} \\ \langle \mathcal{B}_C \rangle &= 2^{\lfloor N \backslash 2 \rfloor} \end{aligned}$$

Now for an arbitrary $\left|H_N^3\right\rangle$:

$$\begin{aligned} \langle \mathcal{B}_Q \rangle &\geq 2^{N-2} - \frac{1}{2} \quad \text{HG state} \\ \langle \mathcal{B}_C \rangle &= 2^{\lfloor N \backslash 2 \rfloor} \end{aligned}$$

M. Gachechiladze, C. Budroni, Otfried Gühne, arXiv:1507.03570 (2015)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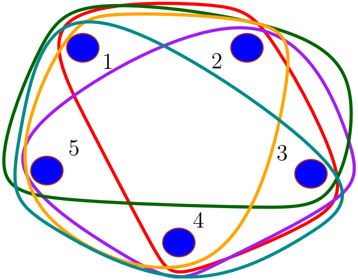
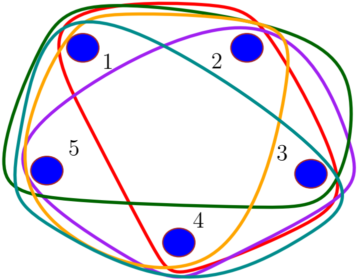
Advantages of hypergraph states



- GHZ state becomes a fully separable state after tracing out one particle.

Advantages of hypergraph states



- GHZ state becomes a fully separable state after tracing out one particle.
- **4-uniform hypergraph states still violate Mermin inequality exponentially!**

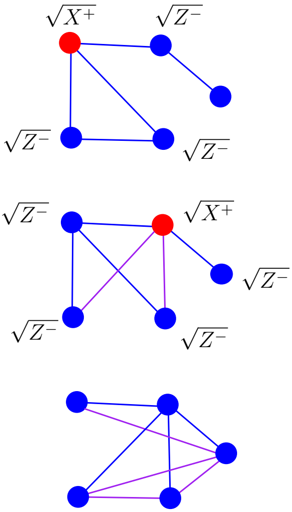
Direct Applications

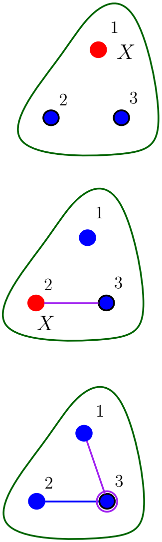
- Heisenberg-limited metrology†
  - an advantage of being more robust to noise;
- Nonadaptive measurement based quantum computation with linear side-processing††

† W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Gühne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, and J.-W. Pan, Nature Phys. 6, 331 (2010).
†† M. J. Hoban, E. T. Campbell, K. Loukopoulos, and D. E. Browne, New J. Phys. 13, 023014 (2011).

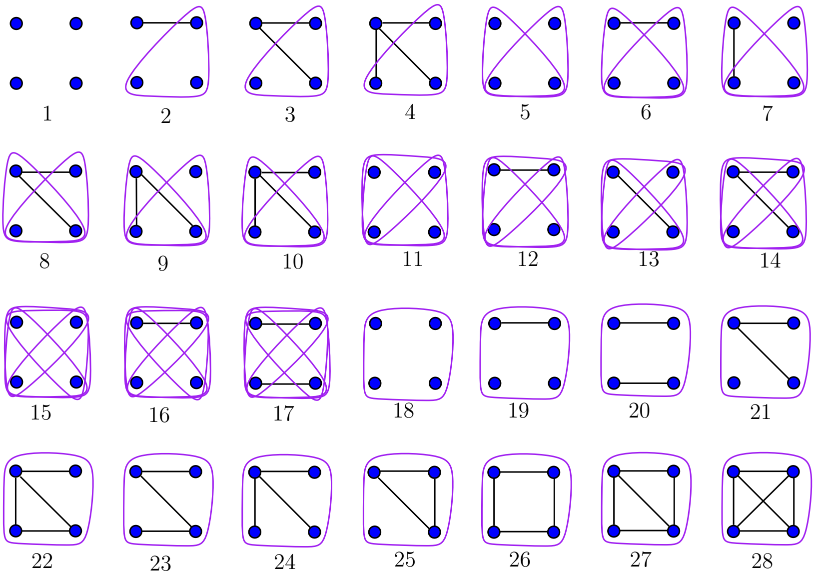
Local Unitary Equivalence Classes

LU in graph and hypergraph states





M. Hein, W. Dür, J. Eisert, R. Raussendorf, M. Van den Nest, H.-J. Briegel arXiv:quant-ph/0602096



O. Gühne, M. Cuquet, F.E.S. Steinhoff, T. Moroder, M. Rossi, D. Bruß, B. Kraus, C. Macchiavello, J. Phys. A: Math. Theor. 47, 335303 (2014)

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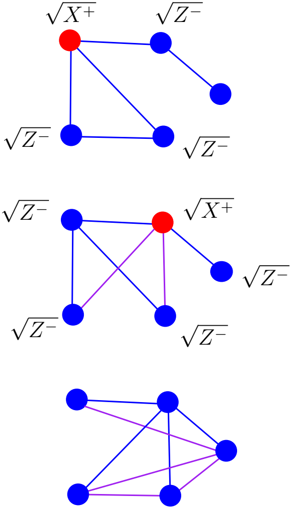

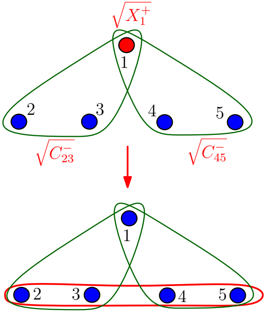

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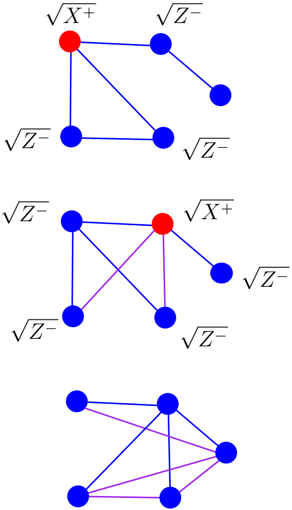

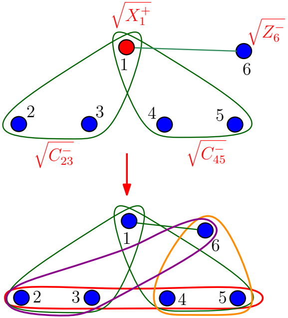

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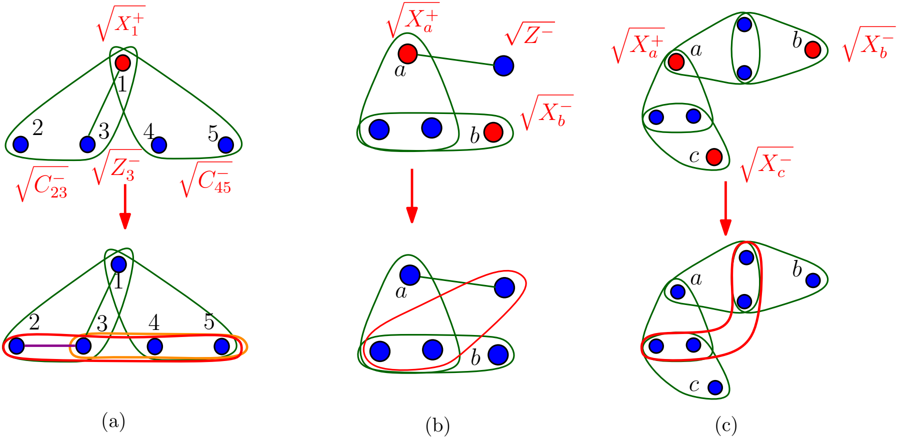
Local Clifford equivalence in hypergraph states



M. Gachechiladze, N.Tsimakuridze, O. Gühne

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- Learn if other hypergraph states are nice as well

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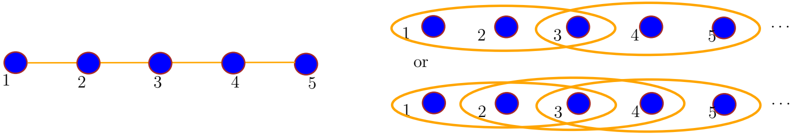

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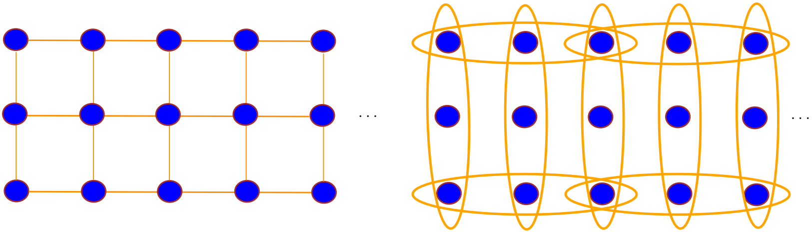

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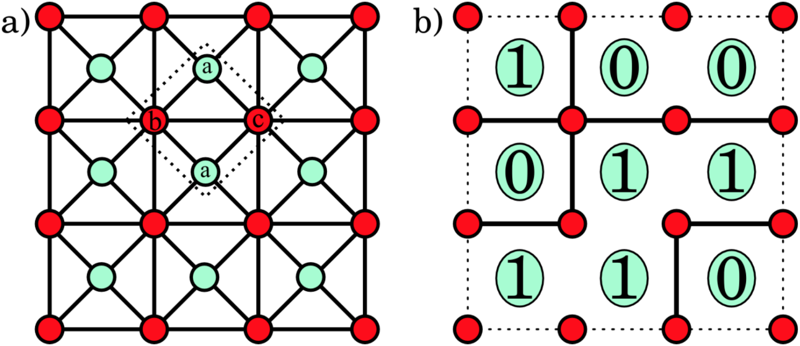
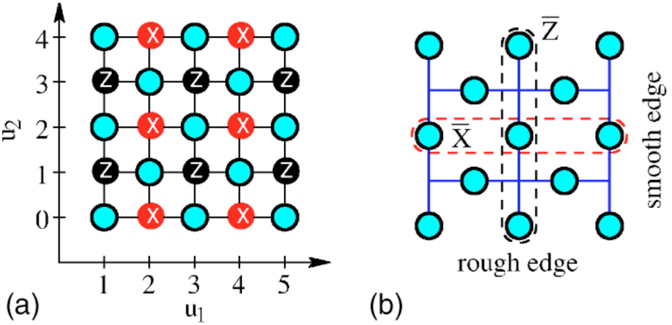
Measurement Based Quantum Computation?



Jacob Miller, Akimasa Miyake, Quantum computation on domain walls of a two-dimensional symmetry-protected topological order arXiv:1508.02695v1

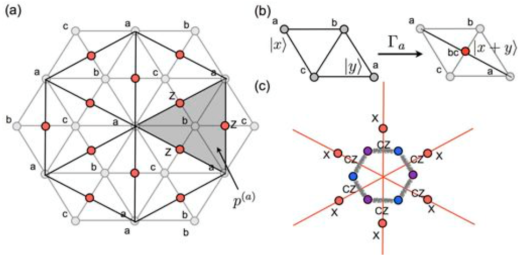
Topological Quantum Memory? Hypertoric Codes



$$H = -\sum_{v} A(v) - \sum_{p} B(p) \qquad (1)$$

R. Raussendorf, S. Bravyi, and J. Harrington, Long-range quantum entanglement in noisy cluster states Phys. Rev. A 71, 062313 (2005)

Topological Quantum Memory? Hypertoric Codes



$$\hat{H} = -\sum_{p \in P^{(a)}, P^{(b)}, P^{(c)}} B_p - \sum_{v^{(a)} \in V^{(a)}} A_{v^{(a)}} - \sum_{v^{(b)} \in V^{(b)}} A_{v^{(b)}} - \sum_{v^{(c)} \in V^{(c)}} A_{v^{(c)}}. \qquad (32)$$

The Hamiltonian can be viewed as three copies of the toric code which are intricately coupled with each other via CZ phase operators.

Beni Yoshida, Topological phases with generalized global symmetries arXiv:1508.03468v1

Topological Quantum Memory? Hypertoric Codes

Thank you!